Economic Optimization of Project Risk Management Efforts

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Statement of Scope and Purpose

Risk management in projects has become an important topic in project management literature. Large projects consist of thousands of work elements involving hundreds of employees, and therefore, risk management in such projects requires structured models, which can be implemented in software. Ben-David and Raz[1] developed a model that captures the interactions among work packages within a project in respect to risks and risk abatement efforts. The purpose of this paper is to extend their work by providing a mathematical formulation that facilitates computer implementation of the model, and by adding further features. Since the risk abatement actions selection problem is a complex problem by nature, we propose a branch and bound optimal algorithm and two heuristic algorithms. In addition, we conduct a performance benchmarking experiment among the three procedures.

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Abstract

Risk exposure reduction is a recognized stage in project planning. The relevant literature mainly comprises of conceptual models that capture effects of uncertainty on projects, and strategies to reduce the level of risk. Ben-David and Raz[1] proposed a generic model that describes the risk abatement actions selection problem. The model opts to allocate risk abatement efforts in the planning stage of a project by integrating the project work breakdown structure (WBS) with the risks generation and effects phenomena. This is achieved by explicitly relating between the project potential risk events, and the impacts of the obtainable risk abatement actions. We extend their work by mathematically formulating their generic model, by adding modeling features (feasibility constraints) and by treating the relevant optimization issues. We solve the problem with an optimal non-efficient branch and bound algorithm and benchmark two alternative efficient heuristic procedures. A wide numerical experiment indicated that the greedy heuristic derived optimal solutions in 92% of the unconstrained cases and in 73% of the constrained cases.

Keywords: Project Management, Risk, Heuristics, and Optimization.

Introduction

Risk management has developed into a standard part of the planning stage of projects - see Buchan[2], PMI[3], Higuera and Haimes[4], Kahkonen[5], and Chapman[6]. For conceptual strategies of dealing with risk see PMI[3], and Kahkonen[5]. Choosing among feasible risk abatement actions has been addressed in several papers: Chapman[7,8], Chapman et al.[9], and Chapman and Cooper[10] applied a semi-Markovian process to model the impact of risk events on the execution of a project, and used Monte-Carlo simulation for evaluating actions. Klein[11] suggests evaluating actions on three project success criteria: duration, cost, and quality. Some of the recent work on project risk analysis has utilized the WBS as the basis for risk identification (e.g. Leung et al.[12] and Rao Tummala and Burchett[13]). Following this approach, Ben-David and Raz[1] have developed a model that integrates the project's scope into the risk management process, and allows focusing on causes and effects of risks as they are distributed among the project activities. Their approach explicitly relates between the activities of the project and the set of risk abatement actions available for implementation. In addition, it recognizes that the implementation of a risk abatement action can cause secondary risks, and that the combined effects of several risk abatement actions may differ from the sum of their individual effects. In this paper we extend that work in three ways. First, we provide a complete mathematical formulation of the model and of the actions selection problem, including extensions that allow for feasibility constraints among risk abatement actions. The formulation is essential for the design of decision-support systems for large-scale projects. Second, we present optimal and heuristic algorithms for solving the risk abatement selection problem. Finally, we report the results of an experiment benchmarking the performance of these algorithms. The results indicate that solutions very close to the optimal can be obtained with a greedy algorithm requiring modest computational resources.

Model Formulation

The objective of the Ben-David and Raz[1] model is to find the economically optimal combination of risk abatement actions. It relates among three types of entities: project work elements, risk events that work elements are exposed to, and a set of possible risk abatement actions that the decision-maker can select from.

Work elements

The project work elements are all the components of the WBS. We define $w = 1, 2,...,W$ as the set **W** of work elements. Since the WBS is a hierarchical representation of the project's tasks, the members of the set **W** are not mutually exclusive. In this manner it is possible to relate risks to intermediate WBS elements and not only to the lowest level elements.

Risk events

When a risk event materializes it affects some work elements of the project. The risk events are dichotomous (multilevel events can be treated as a number of mutually exclusive dichotomous events, one for each level). We define $r = 1, 2,...,R$ as the set **R** of risk events. The model addresses three attributes of risk events: source, probability of occurrence, and impact.

Risk Sources

Each risk event has a single source. Internal sources are work elements of the project, upon which we have some control, and are distinguished from external sources of risk events (e.g. weather, economy). We define $s = 1, 2,...,S$ as the set **S** of sources, where the first W sources are the work elements and the remaining $S - W$ are external. A risk source can generate multiple risk events, however, a risk event is generated by a single source.

Risk probability

The probability of occurrence of a risk event depends on its source. We define a probability matrix **P** where its element $p_{r,s}$ is the probability that source s will cause a risk event r.

Risk impacts

The occurrence of a risk event may impact one or more work elements of the project. We define an impact matrix **M** where its element $m_{r,w}$ is a monetary loss to work element w caused by risk event r.

Risk abatement actions

Risk abatement actions modify the probability and/or the impact of risk events. Some generic examples of risk abatement actions include: reducing or otherwise changing the scope of a work element; adding resources to a work element; using a different type or combination of resources; subcontracting a work element. We define $a = 1, 2,...,A$ as the set **A** of risk abatement actions; X_a as the selection decision variable of action a, where $X_a = 1$ indicates selection and 0 otherwise; c_a as the abatement action cost; $v_{r,w,a}$ as the effect factor of action a on the probability of risk r originated from work element w, where $\mathbf{v}_{r,w} = (v_{r,w,1},\ldots,v_{r,w,A})$; $u_{r,s,a}$ as the effect of action a on the impact of risk r originated from risk source s where $\mathbf{u}_{r,s} = (u_{r,s,1},\ldots,u_{r,s,A})$. If an effect attribute ($v_{r,w,a}$ or $u_{r,s,a}$) is zero then it has no effect. Let $\mathbf{X}_{(\mathrm{AXA})}$ be a diagonal matrix with $X_{a,a} = 1$ if action a is chosen and 0 otherwise. Then $\mathbf{X}\ \mathbf{v}_{r,w}$ and $\mathbf{X}\ \mathbf{u}_{r,s}$ are the resulted probability and impact effect vectors with the chosen actions. In general, the modified probability of risk event r from source w is given by $f(p_{r,w}\ , \mathbf{X}\ \mathbf{v}_{r,w})$. And the modified impact of the risk event r from source s is given by $h(m_{r,s}\ , \mathbf{X}\ \mathbf{u}_{r,s})$. The number of arguments each of the functions f and h gets is $A + 1$. The structures of these functions will be discussed later.

Main Assumptions

1. The risk abatement selection problem is a static decision problem. If a project involves several milestones review then a new static problem can be solved in each review.
2. The risk events are mutually independent. If probabilities were correlated, then one would create *event bundles* that contain all correlated events and treat each as a single event.
3. A work element can be a source of risk events and it may also be exposed to risk events that originate at other risk sources.
4. Work elements may be affected positively as well as negatively, by implementation of risk abatement actions.
5. A risk abatement action cannot affect the probability of a risk event that originates from external sources; it can however affect its impacts.

Objective function

We ignore the baseline cost of the project, as it was planned prior to any risk abatement action. The total expected costs (TEC), which are risk related, consist of two components: abatement actions costs (AAC), and expected risks loss (ERL). With this definition:

$$\mathrm{AAC}(\mathbf{X}) = \sum_{a=1}^{A} c_a X_{a,a} = \mathbf{cXe}. \qquad [1]$$

Where $\mathbf{c}$ is a row vector of c_a and $\mathbf{e}$ is a column vector of appropriate size of 1's. Clearly, if no abatement action were chosen then:

$$\mathrm{ERL} = \sum_{r=1}^{R} \Big(\sum_{s=1}^{S} p_{r,s}\Big)\Big(\sum_{w=1}^{W} m_{r,w}\Big) = (\mathbf{Pe})(\mathbf{Me})'. \qquad [2]$$

Where ′ is matrix transpose operator. Equation [2] sums up, over all the risk events, the product of probability times overall impact (i.e. monetary loss) on the project. Since each risk originates from a single source, at most one component is positive in the middle summation. The right summation is the total impact of risk r on the project. Given that some actions have been selected, then the expected risks loss becomes:

$$\text{ERL}(\mathbf{X}) = \sum_{r=1}^{R} \left(\sum_{s=1}^{W} f(p_{r,w}, \mathbf{X}\,\mathbf{v}_{r,w}) + \sum_{s=W+1}^{S} p_{r,s}\right)\left(\sum_{w=1}^{W} h(m_{r,s}, \mathbf{X}\,\mathbf{u}_{r,s})\right)$$

$$= (\mathbf{f}(\mathbf{P}, \mathbf{Xv})\mathbf{e})\,(\mathbf{h}(\mathbf{M}, \mathbf{Xu})\mathbf{e})'. \qquad [3]$$

Where **f** and **h** are matrix versions of the function f and h. By assumption 4, abatement actions cannot affect probabilities of risks originated from external sources; hence, the first component of the product is split into W modifiable probabilities and the remaining $(S-W)$, that are not. In the matrix representation these two components are combined with $\mathbf{v}_{r,s} = \mathbf{0}$ for $s = W + 1,\ldots,S$.

Actions Constraints

Often there are logical constraints that limit the combinations of actions that can be selected. For instance, if we chose to exclude a certain task from the project plan due to its high risk, then it would not be possible to select actions that involve alternative resources or technologies for this task. The model allows two types of pairwise constraints: *exclusion*, as illustrated in the preceding example, and *implication*, which means that the selection of one action requires that another specific action be selected too. We define $q_{i,j} = 1$ if actions i and j exclude each other, and $b_{i,j} = 1$ if selection of action i implies selection of action j, and 0 otherwise.

The problem

The general project risk management problem can now be expressed as:

$$\textit{Minimize}\ \text{TEC}(\mathbf{X}) = \text{AAC}(\mathbf{X}) + \text{ERL}(\mathbf{X}), \qquad [4]$$

Subject to:

$$X_{i,i} + X_{j,j} \le 1, \qquad \forall q_{i,j} = 1,\ i, j \in \mathbf{A}, \qquad [5]$$

$$X_{i,i} \le X_{j,j}, \qquad \forall b_{i,j} = 1,\ i, j \in \mathbf{A}, \qquad [6]$$

$$X_{i,i} \in \{0, 1\}, \qquad \forall i \in \mathbf{A}. \qquad [7]$$

Inequalities [5] are the exclusion constraints between pairs of actions. Inequalities [6] guarantee that selection of action i implies selection of action j as well. Optionally, a budget constraint can be included on the abatement actions spending: AAC($\mathbf{X}$) ≤ B. Further extension of the model may include a weighted sum of AAC($\mathbf{X}$) and ERL($\mathbf{X}$) to reflect risk preferences as proposed by Williams[14] and others.

Desired properties

A key issue in the design of the model was how to represent mathematically the effects of the risk abatement actions on the risk properties. The arbitrary order of the abatement actions in A should not affect their combined result. Operators that support this order independence property on a list of arguments are: minimum, maximum, product, summation, and any function on any combination of these operators. The order independence property provides also a significant solution advantage, as it reduces the size of the solution space. When dealing with multiplication operator for modifying probabilities, we should be aware of two points. First, it does not allow for an effect to increase a probability that was initially equal to zero. Second, the effect of actions that increase the probability of a certain risk could in principal, result in probability value that exceeds one. Clearly, both situations are not likely in practice and they can be tested and avoided. The function *Minimum*(1, *Product*(z)), for example, preserves the order independence property and avoids the second problem.

Important Special Case

We will now express ERL for the case when multiplication is used as the modifying function $f(\cdot)$ of the probabilities and minimization is used as the modifying function $h(\cdot)$ of the impacts. The minimum and multiplication operators have the advantage of being simple to comprehend and implement. In formal terms, we assume that:

$$f(p_{r,w}\,,\, \mathbf{X}\,\mathbf{v}_{r,w}) = Product(p_{r,w}\,,\, \mathbf{X}\,\mathbf{v}_{r,w}), \text{ and} \qquad [8]$$

$$h(m_{r,s}\,,\, \mathbf{X}\,\mathbf{u}_{r,s}) = Minimum(m_{r,s}\,,\, \mathbf{X}\,\mathbf{u}_{r,s}). \qquad [9]$$

Meaning that f is the product of its ($A + 1$) arguments and h is the minimum of its ($A + 1$) arguments. The insight underlying these functions is that in both of them the marginal effect of a certain action tends to decline, as the existing value is smaller. In other words, a decreasing marginal benefit, as more actions are included.

Optimal and Heuristic Algorithms

The actions selection problem as stated above is an integer-programming problem. In this section we present three algorithms that solve the problem: an optimal branch and bound algorithm, and two heuristics: a naïve heuristic, based on the principle of maximum net contribution and a greedy heuristic, based on maximum marginal net contribution. All three are suitable for the general model if it preserves the order independence property. We compare them in the numerical experiments under the special case defined by Equations [8] and [9].

Branch and bound algorithm

The branch and bound algorithm starts with a feasible empty set of selected actions, then a binary tree is developed. Each partition level relates to a specific action. At each node at level $a - 1$ of the binary tree, the algorithm divides the solution space into two subsets: a partition that includes solutions with action a ($X_{aa} = 1$), and a partition that includes solutions without this action ($X_{aa}= 0$). A lower bound for the objective function at node θ with $\mathbf{X}^{\theta}$ the already decided actions, is obtained by computing AAC($\mathbf{X}^{\theta}$) + ERL(**I**), where $\mathbf{I}_{(A \times A)}$ is a unity matrix (the expected risk cost when selecting all the not yet decided actions, while ignoring their implementation costs). Partitions that violate the constraints are excluded. The $O(2^{A})$ bounded complexity of the branch and bound method motivated us to derive effective heuristics.

Naïve heuristic

The naïve heuristic is based on the concept of selecting all the actions that individually bring about a net improvement to the value of the objective function, disregarding the interactions among them. Its computational complexity is bounded by $O(A)$. Some solutions may be in infeasible and may require additional adjustments.

Greedy heuristic

The greedy algorithm starts with no selected actions and iteratively evaluates the inclusion of each of the still unselected actions (which satisfy the constraints). The action that results in the greatest positive improvement to the objective function is selected. The process stops when, either none of the remaining actions can improve the objective function, or all the actions have been selected. The computational complexity of the greedy heuristic is bounded by $O(A^{3})$.

Numerical Experiments

Experimental design

In order to evaluate the performance of the heuristics we performed an experiment with simulated data. The factors considered in the experiment were related to the characteristics of the project and to the size of the solution space. Table 1 summarizes the design of the experiment and the way in which data was generated for the experiment. To reduce the complexity of the experiment, each action was associated with one effect, and only one external source of risk was defined.

[Insert Table 1 about here]

Overall there were nine factors, with 1,152 different combinations. For each combination ten problem cases were randomly generated. Each of these 11,520 cases was solved optimally with the branch and bound technique and with the greedy algorithm as described earlier. In addition, the 2,880 unconstrained combinations were solved with the naïve heuristic as well.

Measure of performance

The performance of each heuristic was assessed by the relative closeness of its objective to the one of the optimal branch and bound solution and denoted by Δ:

$$\Delta = \frac{TEC_{Heuristic} - TEC_{Optimum}}{TEC_{No\ Action} - TEC_{Optimum}} \quad [11]$$

A value of zero for Δ means that the heuristic achieved an optimal solution, while larger values indicate worse performance. A value of Δ larger than 100% indicates a solution inferior than the original project with no abatement action.

Results

The average value of Δ for the naïve heuristic (non-constrained cases) was 58%, with the worst result being 712%. For the greedy heuristic (all cases) the average Δ was 1.39%, while the worst result was 69.21%. Table 2 presents the cumulative distribution of Δ for the two heuristics. The performance of the greedy heuristic is clearly superior: it yielded the optimal solution in almost three-quarters of all cases, and in almost 92% of the unconstrained cases, while the naïve heuristic did so in just over 4% of the unconstrained cases. Further, for over 20% of the cases, the naïve heuristic generated a solution worse than the original cost with no risk abatement actions.

[Insert Table 2 about here]

Concluding remarks

Although project risk management is recognized as important, the area lacks widely used analytical tools. The approach presented in Ben-David and Raz[1] explicitly relates between the work contents of the project, the risks, and the selectable abatement actions. The model provides richness not available in previously published models. This includes constraints among the risk abatement actions; representation of secondary or internal risks; risk abatement actions with multiple effects on different work elements affecting either the probability or the damage aspect of risk; and modeling the combined effects of several risk abatement actions. However, it requires further mathematical and algorithmic development. The contributions of this paper are twofold. First, our mathematical formulation facilitates the implementation of this approach to computerized systems required for the programming of large problems. Second, it addresses the problem of complexity posed by solving large-scale projects. An optimal solution to the model can be found using branch and bound. Naïve (ignores interactions among work elements) and Greedy (focuses on marginal contribution of actions) heuristics were compared to the optimal solution in a wide numerical experiment. The greedy heuristic yielded the optimal solution in the majority of the cases, with a small average deviation of about 1.4%.

Extensions to this paper may include multilevel risk events and budget constraints. Also, more project-specific elaborations can be added in the level of the objective function, penalty functions and risk abatement actions interaction functions.

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Tables

Variable	Variable Levels	Data Generation Method
1. Number of risk events	10, 30	
2. Number of work elements	10, 30	
3. Maximum risk event probability	0.1, 0.4, 0.7	The probabilities were drawn randomly from a uniform distribution between 0 and the maximum probability.
4. Damage exposure, implemented as the percentage of zero entries in the impact matrix	10%, 40%, 70%	The actual values for the nonzero entries in the impact matrix were drawn randomly from a uniform distribution between 0 and $10,000.
5. Number of actions that affect a risk probability*	5, 8	The actual values of the effect were drawn randomly from the [0,1] uniform distribution.
6. Number of actions that affect a risk impact*	5, 8	The actual values of the effect were drawn randomly from a uniform distribution between 0 and $10,000.
7. Ratio of implementation cost to expected impact	0.3, 0.7	
8. Number of exclusion constraints*	0, 3	The actions for the constraints were selected randomly
9. Number of implication constraints*	0, 3	The actions for the constraints were selected randomly

* These factors have direct impact on the size of the solution space.

Table 1 – Design of the experiment

Results	Greedy Algorithm (all cases)	Greedy Algorithm (non-constrained cases only)	Naïve Algorithm (non-constrained cases only)
Δ = 0	73.36%	91.84%	4.27%
Δ < 1%	80.59%	95.52%	7.05%
Δ < 2%	84.36%	96.63%	10.63%
Δ < 5%	90.79%	98.02%	18.06%
Δ < 10%	95.28%	98.78%	30.42%
Δ < 20%	98.99%	100.00%	47.12%
Δ < 50%	99.98%	100.00%	65.10%
Δ < 100%	100.00%	100.00%	78.19%
Δ < 200%	100.00%	100.00%	93.89%
Δ < 500%	100.00%	100.00%	99.86%
Δ < 750%	100.00%	100.00%	100.00%

Table 2 – Cumulative distribution of Δ for the two heuristics